Galilean Transformation of the velocity of a Particle:-

Consider two systems S & S', The latter moving with velocity $V = \hat{i} v_x + \hat{j} v_y + \hat{k} v_z$, relative to the former. If $\vec{r}(t)$ and $\vec{r}'(t')$ are the Co-ordinates of any Particle as observed by observer in System S & S'. Then the Galilean transformation equations of Space and Time may be expressed as.

$$\left.\begin{aligned} \vec{r}' &= \vec{r} - \vec{V}t \quad \text{— (A)} \\ \text{and } t' &= t \quad \text{— (B)} \end{aligned}\right\} \quad \text{— (i)}$$

Now differentiating equ^n (1A) with respect to t, we get

$$\frac{dr'}{dt} = \frac{dr}{dt} - V \quad \text{—— (ii)}$$

$$\left[\frac{d\vec{r}'}{dt} = \frac{d}{dt}(\vec{r} - \vec{V}t) = \frac{d\vec{r}}{dt} - \frac{d}{dt}(\vec{V}t) \right]$$

from (1b); we have $dt' = dt$ Therefore equ^n (2) becomes

$$\frac{dr'}{dt'} = \frac{dr}{dt} - V$$

i.e. $$\boxed{\vec{u}' = \vec{u} - \vec{V}} \quad \text{—— (3)}$$

where

$u' = \frac{dr'}{dt}$ = velocity vector of Particle in S'.

$u = \frac{dr}{dt}$ = velocity vector of Particle in S

Equ^n (3) represents The Galilean transformation of The velocity of The Particle = 0.

Teacher's Signature

Galilean Transformation of The Acceleration of The Particle :-

We have Galilean transformation for velocity of Particle is

$$\vec{u}' = \vec{u} - \vec{v}$$

Differentiating above equⁿ with respect to t, we get

$$\frac{d\vec{u}'}{dt} = \frac{d}{dt}(\vec{u} - \vec{v})$$

$$= \frac{d\vec{u}}{dt} = \frac{d\vec{v}}{dt} = 0$$

or $$\frac{d\vec{u}'}{dt} = \frac{d\vec{u}}{dt}$$

or $$\frac{d\vec{u}'}{dt'} = \frac{d\vec{u}}{dt}$$

since $dt' = dt$

or $a' = a$

where $\vec{a}' = \frac{d\vec{u}'}{dt}$ = Rate of change of velocity of Particle in system s'.

= acceleration of The Particle in s'.

and $\vec{a} = \frac{d\vec{u}}{dt}$ = Rate of change of velocity of Particle in system s.

= acceleration of The Particle in system s.

Thus The acceleration observed by The observer in different inertial frames is same, i.e; acceleration is invariant under Galilean transformations.

As mass is also invariant, so

$$\vec{F} = m\vec{a}$$

is invariant in inertial frames.

In other words we may say That Newton's second law is valid in every inertial system i.e. it is invariant under Galilean transformation.

Problem

1.) Show That Newtonian fundamental equ^n are in-Variant under Galilean transformation.

Sol: we Show This by taking Newton's second law of Motion.

In The absolute system of co-ordinates, The Particle acted upon by a force F having an acceleration $\frac{d^2x}{dt^2}$ is given by

$$F = m\frac{d^2x}{dt^2} \quad \text{—— (i)}$$

Now according to Galilean transformation

$$u'_x = u_x - V$$

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

Differentiating equ^n, we get

$$\frac{d^2x'}{dt'^2} = \frac{d^2x}{dt^2} \quad \text{. since } v \text{ is constant.}$$

In Newtonian Mechanics since forces and masses are absolute quantities.

so $F = F'$

$m = m'$

Therefore,

$$F' = m'\frac{d^2x'}{dt^2}$$

i.e. with respect to other frame of reference The second law has The same form i.e. Second law is valid for every inertial system.

Thus we may say that Newtonian fundamental equations are invariant under Galilean transformations.

Problem-2 A ball has velocity $(4\hat{i}-5\hat{j}+10K)$ m/sec. relative to a train moving with velocity $(3\hat{i}+4\hat{j})$ m/sec. relative to an observer on the ground. Calculate the velocity of the ball relative to ground.

Sol:- Consider the train to be system S' and the ground the system S. Then we have

$$V = (3\hat{i}+4\hat{j}) \text{ m/sec}$$

and

$$u' = (4\hat{i}-5\hat{j}+10K) \text{ m/sec}$$

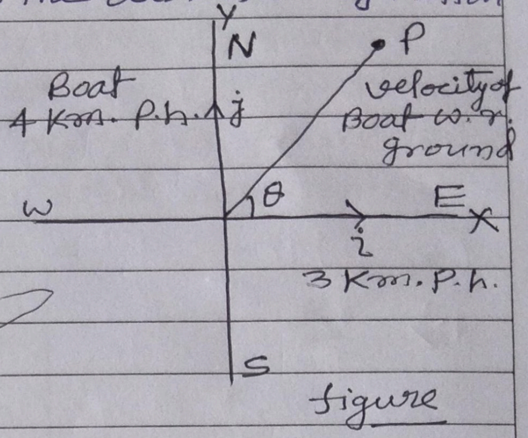
$\therefore$ The velocity of the ball relative to ground is given by

$$\begin{aligned} u &= u' + V \\ &= (4\hat{i}-5\hat{j}+10K) + (3\hat{i}+4\hat{j}) \\ &= 7\hat{i} - \hat{j} + 10K \text{ m/sec} \end{aligned}$$

Problem-3 Water in a river moves east at 3 Km. p.h and a boat heads north at 4 Km. p.h. w.r.t. water. Find the velocity of the boat w.r.t. ground and also find the direction

Sol:- ~~Con~~ ~~Take the~~ Take east along +ve direction of x-axis and north along +ve direction of y-axis.

OX denotes the direction of motion of flow of water w.r.t. ground and OY as the direction of motion of boat w.r.t. water.



Now we can consider two frames of reference say S & S' one on ground and other on water. The frame in water may be considered to move w.r.t. that on ground with a velocity 4 Kg.P.h., so that the velocity of boat w.r.t. the frame of reference S (fixed in water) = 4 Kg. P.h. in north direction

i.e; $u' = 4\hat{j}$ Km.P.h.

where $\hat{j}$ is unit vector in the direction of y axis in frame S'.

The Motion of boat w.r.t. ground is vector sum of motion of boat w.r.t. water u' and the Motion of Water w.r.t. ground V.

we have, $V = 3\hat{i}$ Km. P.h.

$\therefore$ from eqn

$$u' = u - v$$

$$u = u' + V = (4\hat{j} + 3\hat{i}) \text{ Km. P.h.}$$

or $u = |u| = \sqrt{(4^2 + 3^2)}$

$= \sqrt{25} = 5$ Km. P.h.

$$\tan\theta = \frac{4}{3} = 1.333$$

$$\therefore \theta = \tan^{-1} 1.333 = \tan^{-1}\frac{4}{3}$$

That is the boat has a velocity of 5 Km P.h. w.r.t. ground making an angle of $\left(\tan^{-1}\frac{4}{3}\right)$ from east.